



Date: 07-11-2024

Dept. No.

Max. : 100 Marks

Time: 09:00 am-12:00 pm

SECTION A

Answer ANY FOUR of the following

4 x 10 = 40 Marks

1. Define stochastic process and explain its types.
2. Show that communication is an equivalence relation.
3. Explain spatially homogenous Markov chains.
4. State and prove the necessary and sufficient condition for a state i to be recurrent.
5. Derive the differential equations for a pure birth process.
6. Explain type I and type II counter models with the necessary diagrams.
7. Let $\{X_n, n \geq 0\}$ be a Markov chain with state space $S = \{0, 1, 2, 3\}$ and the transition probabilities be
 $P_{00} = 1/6$, $P_{01} = 1/3$, $P_{02} = 1/2$, $P_{10} = P_{11} = 1/2$, $P_{20} = 1/6$, $P_{21} = 1/3$, $P_{22} = 1/2$,
 $P_{31} = 1/6$, $P_{32} = 1/3$, $P_{33} = 1/2$.
Find the stationary probability distribution.
8. Narrate branching process with the help of two examples.

SECTION B

Answer ANY THREE of the following

3 x 20 = 60 Marks

9. Show that the three-dimensional random walk is transient.
10. State the postulates of Poisson process and derive $P_n(t)$ for it.
11. Derive backward and forward Kolmogorov differential equations for birth and death process.
12. State and prove the elementary renewal theorem.
13. Let a Markov chain on the states $\{0, 1, 2, 3, 4, 5\}$ has the following one-step transition probabilities:
 $P_{00} = 1$, $P_{11} = 3/4$, $P_{12} = 1/4$, $P_{21} = 1/8$, $P_{22} = 7/8$, $P_{30} = P_{31} = 1/4$, $P_{33} = 1/8$,
 $P_{34} = 3/8$, $P_{40} = 1/3$, $P_{42} = P_{43} = 1/6$, $P_{44} = 1/3$, $P_{55} = 1$.
(a) Find the equivalence classes. (b) Find period for different classes.
(c) Show that the states 0, 1, 2, 5 are recurrent and 3, 4 are transient.
(3+3+14)
14. Establish the probability generating function relation and find mean and variance for branching process.

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